Charged vortices in superfluid systems with pairing of spatially separated carriers

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It is shown that in a magnetic field the vortices in superfluid electron-hole systems carry a real electrical charge. The charge value depends on the relation between the magnetic length ℓ_B and the Bohr radiuses of electrons a_B^e and holes a_B^h. In double layer systems at filling factors $\nu_e = \nu_h = \nu$ and for $a_B^e, a_B^h \gg \ell_B$ the vortex charge is equal to the universal value νe .

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It is generally believed, that the vortices in superconductors are connected with an applied magnetic field, while the magnetic field does not have any influence on the properties of the vortices in electrically neutral superfluid systems. The aim of this letter is to show that in superfluid systems subjected by a magnetic field the vortices will have a real electrical charge (the compensating charge of the opposite sign will appear on the surface of the system). In general case the charge of the vortices takes a fractional value. For the first time the fractional charge of the vortices was predicted by Laughlin [1] for the two-dimensional electron gas in a quantized magnetic field. Then, it was established in Ref. [2, 3], that in double layer electron systems with the half-filling of the lowest Landau levels in each layer the vortices should carry the charge equals to $\pm e/2$ (here and below e is the absolute value of the electron charge).

We will show that in any superfluid system the magnetic field results in an appearance of the vortex charge proportional to the polarizability of the particles and inverse proportional to their effective mass. The estimates show that for reachable values of magnetic fields the vortex charge is unobservable small one in superfluid phases of He isotopes and in Bose gases of alkali metals, while it can be of order of the electron charge in superfluid systems with pairing of spatially separated electrons and holes.

The authors of Ref. [4] call the possibility of the electron-hole pair superfluidity in question based on the fact that the interband transitions fix the phase of the order parameter and result in a transition into a dielectric state. But it was established in Refs. [5, 6] that the interdiction on the electron-hole superfluidity can be removed in systems, where the spatially separated electrons and holes are coupled. In these systems the interband transitions coincide with the interlayer ones and usually they are exponentially small. The superfluid state of the pairs with spatially separated components has both the superfluid and superconducting features. The superfuid flow of such electron-hole pairs is accompanied with real supercurrents flowing in opposite directions. Therefore, we will call these systems the condenser superconductors.

In Refs. [5, 6] the pairing of a conducting band electron from the one layer with a valence band hole from the other layer was considered. Then in a number of theoretical papers [2, 3, 7, 8, 9, 10] it was shown a possibility of superfluidity of pairs composed from spatially separated electrons and holes belonging to the conducting band. This possibility is realized in double layer electron systems in a magnetic field normal to the layers for the case of the total filling factor $\nu_T = \nu_1 + \nu_2 = 1$. During almost 10 years there were many efforts to observe the condenser superconductivity experimentally [11, 12, 13, 14]. Now it seems that these effort have been crowned with success [15, 16].

The principal result can be obtained from general consideration which does not imply a concrete form of the Hamiltonian. Let us consider a superfluid system subjected with crossed electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields. Let $\mathcal{E}$ is the density of the energy of the system, and $\mathbf{\Pi}$ is the density of its generalized momentum. Note, that the values of $\mathcal{E}$ and $\mathbf{\Pi}$ do not include the energy and the momentum of the external fields that polarize the system. We are interesting in a relation between the momentum density $\mathbf{\Pi}$ and the dipole momentum density $\mathbf{P}$. To establish this relation it is convenient to consider the frame of reference, in which the electric field is equal to zero. The velocity of that frame relative to the lab frame is equal to

$$\mathbf{u} = c\frac{\mathbf{E}\times\mathbf{B}}{B^2}. \tag{1}$$

The expression (1) is valid to within the linear order of the factor u/c. The condition $u/c \ll 1$ is satisfied at $E^2 \ll B^2$. Below we assume this condition is fulfilled.

Let us introduce the notation $\mathcal{E}_0$ and $\mathbf{\Pi}_0$, the energy density and the momentum density in the new frame, correspondingly. The relation between the energies and the momenta in the lab frame and in the new one can be obtained from the transformations

$$\mathbf{\Pi} = \mathbf{\Pi}_0 + \rho\mathbf{u}, \tag{2}$$

$$\mathcal{E} = \mathcal{E}_0(\mathbf{\Pi}_0) + \mathbf{\Pi}_0\mathbf{u} + \frac{\rho u^2}{2}. \tag{3}$$

Here ρ is the mass density. Expressing the momentum $\mathbf{\Pi}_0$ in Eq. (3) in terms of the momentum $\mathbf{\Pi}$ we obtain

$$\mathcal{E} = \mathcal{E}_0(\mathbf{\Pi} - \rho\mathbf{u}) + \mathbf{\Pi}\mathbf{u} - \frac{\rho u^2}{2}. \tag{4}$$

Then, we take into account that

$$\frac{\partial \mathcal{E}}{\partial \mathbf{\Pi}} = \mathbf{v}, \qquad \frac{\partial \mathcal{E}}{\partial \mathbf{E}} = -\mathbf{P}, \tag{5}$$

where $\mathbf{v}$ is the velocity of the superfluid system. Using Eqs. (4) and (5), we find the required relation

$$\mathbf{\Pi} = \rho\mathbf{v} - \frac{1}{c}\mathbf{P}\times\mathbf{B}. \tag{6}$$

In general case the kinematic momentum $\rho\mathbf{v}$ is of order of the generalized momentum $\mathbf{\Pi}$. But, as it is shown below, for the condenser superconductors in a strong magnetic field the kinematic momentum is much smaller than the generalized one. Consequently, in a strong magnetic field the term $\rho\mathbf{v}$ in Eq. (6) can be omitted. Taking into account, that in the superfluid system $\mathbf{\Pi} = n\hbar\nabla\varphi$, where n is the density of the particles (we consider the temperature $T = 0$), and φ is the phase of the order parameter, we obtain from Eq. (6) the following relation

$$n\hbar\nabla\varphi = -\frac{1}{c}\mathbf{P}\times\mathbf{B}. \tag{7}$$

Let the system, lying in the (x, y) plane, is subjected with a uniform magnetic field, directed along the z axis. Taking the curl of the both parts of Eq. (7), we obtain

$$n\hbar\,\mathrm{curl}_z\nabla\varphi = \frac{H}{c}\mathrm{div}_2\mathbf{P}, \tag{8}$$

where $\mathrm{div}_2\mathbf{P}$ is the two-dimensional divergence. of the vector $\mathbf{P}$. This quantity taken with its sign changed is equal to the polarization charge density $\rho_{\rm pol}$. On the other hand, the left hand side of Eq. (8) is nonzero only in case, when the quantized vortices exist in the system. Then

$$\mathrm{curl}_z\nabla\varphi = 2\pi\sum_i \delta(\mathbf{r}-\mathbf{r}_i)n_i, \tag{9}$$

where $n_i = \pm 1$ the upper (low) sign corresponds to the vortices rotating in the counter-clockwise (clockwise) direction, and the summation is over the vortex centers. Integrating the both parts of Eq. (8) over an arbitrary area, for vortices of the same sign one finds

$$\pm\, 2\pi\hbar n N_v = \frac{H}{c}\int \rho_{\rm pol} dS \equiv \frac{H}{c}Q_v. \tag{10}$$

Here N_v is the number of the vortices in the area S, and Q_v is their charge. It follows from this, that vortex charge is equal to

$$q \equiv \frac{Q_v}{N_v} = \pm 2\pi\frac{\hbar c}{B}n = \pm 2\pi e\ell_B^2 n. \tag{11}$$

In the case of the electron-hole pairing in the lowest Landau level, the density of the pairs n is related with the filling factors $\nu_e = \nu_h = \nu$ by the formula $\nu = 2\pi\ell_B^2 n$. Thus, in strong magnetic fields in the superfluid phase the quantized vortices carry an electrical charge $q = \pm\nu e$.

To clarify how general is the result obtained let us analyze the behavior of the condenser superconductor in a strong magnetic field perpendicular to the layers at small filling factors ν, when the electron-hole pair gas can be considered as a weakly interacting Bose gas. Let us consider two 2D conducting layers separated by the dielectric layer of the width d with the electron carriers in one layer and the hole carriers in the adjacent layer. We consider $m_h \gg m_e$. In a strong magnetic field the large difference of the masses m_e and m_h may result in a situation, when the electron Bohr radius $a_B^e = \varepsilon\hbar^2/m_e e^2$ is much larger then the magnetic length $\ell_B = (c\hbar/eB)^{1/2}$, and the hole Bohr radius $a_B^h = \varepsilon\hbar^2/m_h e^2$ can be smaller or larger then ℓ_B. In this case the energy spectrum of the bounded electron-hole pair, which is formed due to the Coulomb interaction between spatially separated carriers, was found in [17]. The part of the pair energy depending on the momentum of the pair $\boldsymbol{\pi}$ and the velocity $\mathbf{u}$ is equal to

$$\Delta\mathcal{E} = \frac{\pi^2}{2M_*} + \frac{M_B}{M_*}\mathbf{u}\boldsymbol{\pi} - \frac{1}{2}\frac{M_B}{M_*}m_h u^2. \tag{12}$$

Here $M_* = M_B + m_h$, the pair effective mass, M_B, the "magnetic mass"

$$M_B = \frac{4}{\sqrt{2\pi}}\frac{\varepsilon\hbar^2}{e^2\ell_B} = \frac{4}{\sqrt{2\pi}}m_h\frac{a_B^h}{\ell_B}. \tag{13}$$

Introducing the pair polarizability

$$\alpha(B) = M_B\frac{c^2}{B^2} = \frac{4\varepsilon}{\sqrt{2\pi}}\ell_B^3, \tag{14}$$

one can rewrite the correction $\Delta\mathcal{E}$ in the form

$$\Delta\mathcal{E} = \frac{1}{2M_*}\left(\boldsymbol{\pi} + \alpha(B)\frac{\mathbf{E}\times\mathbf{B}}{c}\right)^2 - \frac{\alpha(B)}{2}E^2. \tag{15}$$

Analogous expression was obtained in [18] for an electrically neutral atom in crossed fields for the case of small magnetic fields. In that case Eq. (15) contains the zero magnetic field polarizability of the atom $\alpha(0)$ instead of $\alpha(B)$ and the mass of the atom M instead of the mass of the pair M_*.

Replacing the momentum $\boldsymbol{\pi}$ with the operator $-i\hbar\nabla$, from Eq. (15) we obtain the Hamiltonian of the electron-hole pairs. In the low density limit, when the size of the pair is much smaller then the distance between the pairs and the exchange effects are inessential, the pairs can be considered as true bosons. At low temperatures the rarefied Bose gas should form a superfluid state. The

superfluid phase can be described by the order parameter Ψ. The order parameter satisfies the equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \frac{1}{2M_*}\left(-i\hbar\nabla + \alpha(B)\frac{\mathbf{E}\times\mathbf{B}}{c}\right)^2\Psi - \frac{\alpha(B)}{2}E^2\Psi + \gamma|\Psi|^2\Psi. \qquad (16)$$

The last term in the r.h.s. of Eq.(16) describes the interaction between the pairs. One can show that in the the limit $d \ll \ell_B$ the interaction constant is equal to $\gamma = (\pi/2)^{3/2}e^2d^2/\varepsilon\ell_B$. The vanishing of the interaction constant at $d = 0$ is the consequence of the exact compensation of the Coulomb forces between the pairs (compare with [19]). Presenting the order parameter in the form $\Psi = |\Psi|e^{i\varphi(r)}$, we obtain from Eq.(16) the velocity of the superfluid component

$$\mathbf{v}_s = \frac{1}{M_*}\left(\hbar\nabla\varphi + \alpha(B)\frac{\mathbf{E}\times\mathbf{B}}{\hbar c}\right). \qquad (17)$$

To obtain the dipole momentum of the unit area $\mathbf{P}$ we take into account that the r.h.s. of Eq. (16) is the variational derivative over $\Psi^*(\mathbf{r})$ of the energy functional of the Gunzburg-Landau type for superconductors. The derivative of that functional over the electric field $\mathbf{E}$ taken with the opposite sing is $\mathbf{P}$:

$$\mathbf{P} = \alpha(B)\left[\left(1 - \frac{M_B}{M_*}\right)\mathbf{E} + \frac{\hbar}{M_*c}\nabla\varphi\times\mathbf{B}\right]|\Psi|^2. \qquad (18)$$

The expression (18) can be rewritten in the form

$$\mathbf{P} = \alpha(B)\Big(\mathbf{E} + \frac{1}{c}\mathbf{v}_s\times\mathbf{B}\Big)|\Psi|^2. \qquad (19)$$

This result means that not only the electric field $\mathbf{E}$, but also the Lorentz force polarizes the pair, acting in opposite directions on the positive and negative charges of the pair. However, such a simple expression for $\mathbf{P}$ is valid within the linear accuracy in v_s/c.

One can note that the expressions (17) and (18) are in agreement with Eq. (6). Actually, if we exclude from Eqs. (17) and (18) the electric field $\mathbf{E}$ and take into account that in the case considered $\rho = m_h|\Psi|^2$, we arrive to the relation (6).

The polarization charge ρ_{pol} is found by taking the two-dimensional divergence of the both sides of Eq. (18). To calculate the derivatives in the r.h.s. of Eq. (18) we take into account that the relation (9) for the $\mathrm{curl}_z\nabla\varphi$ takes place. Beside this we omit the terms containing the quantity $\nabla|\Psi|$. We are not interested here in the structure of the vortex core (we consider it as the mathematical point), and out of the core the terms containing $\nabla|\Psi|$ are small at small fields E. It allows to replace $|\Psi|^2$ with the pair density n. Finally, we obtain

$$\rho_{pol}(\mathbf{r}) = -\alpha(B)n\left[\left(1 - \frac{M_B}{M_*}\right)\mathrm{div}_2\mathbf{E} + \sum_i 2\pi\frac{\hbar B}{M_*c}n_i\delta(\mathbf{r}-\mathbf{r}_i)\right]. \qquad (20)$$

Thus, the polarization charge in a superfluid system in a magnetic field can be caused by the polarization of the medium by the electric field with a nonzero divergence, or by the existence of the quantized vortices in the system. It also follows from Eq. (20), that the charge of the i-th vortex is equal to the coefficient of $\delta(\mathbf{r}-\mathbf{r}')$, namely

$$q = \pm 2\pi\frac{\hbar B}{M_*c}\alpha(B)n. \qquad (21)$$

Eq. (21) yields the vortex charge for the electron-hole double layer systems in an arbitrary magnetic fields. The same result is valid for the electron-electron double layer system with the substitution $M_* = M_B + 2m_e$.

In weak magnetic fields ($\ell_B \gg a_B^e$) the polarizability $\alpha = \gamma(a_B^e)^3$, where $\gamma \sim 1$, and the effective mass $M_* \simeq m_h + m_e \simeq m_h$. Then, the vortex charge is equal to

$$q = \pm\frac{2\pi\gamma}{\varepsilon}\left(\frac{a_B^e}{\ell_B}\right)^2 a_B^e a_B^h ne. \qquad (22)$$

In high magnetic fields ($a_B^e \gg \ell_B \gtrsim a_B^h$) using Eq. (14) for $\alpha(B)$ one obtains

$$q = \pm 2\pi\ell_B^2 n\frac{M_B}{M_*}e = \pm\frac{M_B}{M_*}\nu e. \qquad (23)$$

Finally, in ultra high fields ($a_B^h \gg \ell_B$) the effective mass $M_* = M_B(1+\sqrt{2\pi}\ell_B/4a_B^h) \to M_B$ and the vortex charge obeys the universal relation $q = \pm\nu e$.

Let us present here some estimates. In the magnetic field $B = 10$ T the magnetic length $\ell_B \approx 80$ Å. In GaAs heterostuctures with the dielectric constant $\varepsilon = 13$ the magnetic mass is very small ($M_B \approx 10^{-28}$ g). Due to the smallness of this quantity the vortex charge can be of order the electron charge only in superfluid systems with the boson mass of order the electron mass m_0. For the electron-hole double layers at $m_h = 0.4\ m_0$ and $m_e = 0.067m_0$ we obtain $M_B/M_* \approx 0.2$ and the vortex charge is equal to $q \approx 0.2\nu e$. For the electron-electron double layer system with the same m_e we find $M_B/M_* \approx 0.5$ and the vortex charge $q \approx 0.5\nu e$ The derivation of q from the universal value is connected with that the ratio $\ell_B/a_B^e \approx 0.8$ and the strong inequality $\ell_B \ll a_B^E$ does not satisfy in this case.

The number of the vortices and their spatial distribution depends on the electric field $\mathbf{E}$. Integrating the both sides of Eq. (17) along a certain contour, we find

$$M_*\oint \mathbf{v}_s d\mathbf{l} = 2\pi\hbar N_v - \frac{\alpha(B)}{c}B\oint \mathbf{E}\times d\mathbf{l}. \qquad (24)$$

Here N_v is the number of the vortices inside the contour (we consider the vortices of the same vorticity). It follows from Eq.(24), that the velocity $\mathbf{v}_s$ can be reduced

under the appearance of the vortices - in such a way we lower the kinetic energy of the system. When the vortex distribution is considered as a continuous one, the vortex density $n_v(\mathbf{r})$ can be introduced

$$N_v = \int n_v(\mathbf{r})d\mathbf{r}. \tag{25}$$

Putting the r.h.s. of Eq.(24) to zero we find from Eqs. (24) and (25) the relation

$$n_v(\mathbf{r}) = \frac{\alpha(B)B}{2\pi\hbar c}\mathrm{div}_2\mathbf{E}. \tag{26}$$

In the case of a condenser superconductor of a disk shape and for the electric field directed along the disk radius and independent of the angle φ, it is reasonable to assume, that the vortices situated on the circumferences centered at the disk center. If the radius of the i-th circumference is ρ_i and the number of the vortices on it is N_i, then it follows from the definition of $n_v(\mathbf{r})$

$$N_i = \int_{\rho_i}^{\rho_{i+1}} \int_0^{2\pi} n_v(\mathbf{r}) r dr d\theta. \tag{27}$$

This equation determines the relation between the quantities ρ_i and N_i. The values of ρ_i can be found from the assumption that the average distance between the vortices on a given circumference (which is equal $2\pi\rho_i/N_i$) coincides with the average distance ρ_{i+1} - ρ_i between the vortices belonging to the adjacent circumferences. It yields

$$N_i = \frac{2\pi\rho_i}{\rho_{i+1} - \rho_i}. \tag{28}$$

Eqs. (26)-(28) allow to find the values of N_i and ρ_i up to a factor of order of unity.

A macroscopic number of the vortices with equal vorticites can also emerge in the absence of the electric field. It is realized when besides the uniform field $\mathbf{B}_z$ there is an extra field $\mathbf{B}_\tau$ with $\mathrm{div}_2\mathbf{B}_\tau \neq 0$ ($\mathbf{B}_\tau$ is parallel to the plane of the structure). Indeed, one can show (compare with [20]) that in the field $\mathbf{B}_\tau$ the energy of the pair of spatially separated electron and hole is equal to

$$\mathcal{E} = \frac{1}{2M_*}(\boldsymbol{\pi} + \frac{ed}{c}\hat{z} \times \mathbf{B}_\tau)^2. \tag{29}$$

One can see that energy (29) differs from the expressions (15) only by that the induced dipole momentum $\alpha(B)\mathbf{E}$ is replaced with the spontaneous momentum $ed\hat{z}$. Therefore, the dipole momentum of the unit area can be obtained from Eq. (19) replacing the induced momentum with the spontaneous one. Then, taking the divergence of $\mathbf{P}$, we find

$$\rho_{pol}(\mathbf{r}) = -\frac{\alpha(B)Bn}{M_* c}\left[\frac{ed}{c}\mathrm{div}_2\mathbf{B}_\tau + \sum_i 2\pi\hbar n_i \delta(\mathbf{r} - \mathbf{r}_i)\right]. \tag{30}$$

It follows from this expression that in this case the vortex charge is equal the value found above and in the continuous limit the vortex density is $n_v(\mathbf{r}) = (ed/2\pi\hbar c)\mathrm{div}_2\mathbf{B}_\tau$.

At nonzero temperatures the charged vortices will emerge in condenser superconductors in a fluctuation way, in similarity with the same phenomena in a thin He-II film. The circumstance that the vortices are charged does not influence in the first approximation on the thermodynamic features of the system. It is connected with that the Coulomb correction to interaction between the vortices falls down much faster (by the power law) then bare logarithmic interaction between them. The last one, as is well known, results in a Kosterlitz-Thouless transition. Since the sign of the vortex charge is in one to one correspondence with the sign of the vorticity, at temperatures below the Kosterlits-Thouless temperature the vortex-antivortex pairs should be electrically neutral. At temperature above the Kosterlits-Thouless the vortices and antivortices decouple, and free electrical charges appear. It reveals itself in a principal change of the conducting properties of the system under the phase transition.

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